Book Review
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Oliver Stein (2003) Bi-level Strategies in Semi-infinite Programming. Kluwer,
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This is the first monograph which extensively treats theory and numerical aspects of so-called *generalized semi-infinite programs* (GSIP): minimize f on $M = \{x \in X | g_i(x, y) \leq 0 \ \forall y \in Y(x), i \in I\}$, where I is a finite set, $X \subset \mathbb{R}^n$ is the solution set of a finite system of equations, and the index sets $Y(x) \subset \mathbb{R}^m$ may be of infinite cardinality. The mapping $Y(\cdot)$ is supposed to be closed and locally bounded. It is often described by finitely many nonlinear equations and inequalities, and all data functions are considered to be sufficiently smooth. If $Y(x) \equiv Y$ for some nonempty compact set Y, GSIP reduces to a usual *semi-infinite program* (SIP); this is a model which has been extensively studied since the 1960ies.

A standard idea in semi-infinite programming consists in reformulating the problem as a bi-level optimization problem by writing $M = \{x | \varphi_i(x) \leq 0, i \in I\}$, where φ_i is defined by $\varphi_i(x) = \sup_{y \in Y(x)} g_i(x, y)$, i.e., it is the optimal value function of a parametric nonlinear program, provided that $Y(x)$ is given by smooth equations and inequalities. Powerful results from parametric optimization then lead to optimality conditions and decomposition techniques. In the spirit of this idea, the author systematically exploits the bi-level structure of GSIP. The study benefits from the cooperation with H.Th. Jongen (and some of his former students) who has been a pioneer in structural analysis of optimization problems.

One of the author's main messages is to point out that the class of GSIP is much more complicated than that of usual SIP, though in some cases an equivalent formulation holds true. In fact (see Chapter 2), complex problems like reverse Chebyshev approximation, certain robust optimization models, and disjunctive programming motivate to investigate this class. The author provides a concise structural analysis of feasible sets of GSIP and shows that these sets can possess a topological structure which is not known from finite programs or standard SIP (see Chapter 3). Chapter 4 of the book is concerned with first order optimality conditions, including the discussion of general constraint qualifications, criteria of Kuhn-Tucker and John type, respectively, and the case of degenerate $Y(x)$. In Chapter 5, a conceptually new solution method for GSIP is proposed. It utilizes the bi-level structure: first a GSIP is transformed via a Stackelberg game into a mathematical program with equilibrium constraints (MPEC), then the MPEC is regularized by certain NCP functions. The method is of continuation type and replaces GSIP by a sequence of (finite) nonlinear programs. Chapter 6 presents first numerical experiences for an application to design centering.

All at all, the book highlights very well recent developments in a field of active research, though interesting topics like sensitivity analysis and second-order growth conditions for GSIP are beyond the scope of this book. The material is well presented, preliminaries are discussed in detail, and many illustrations help to understand the complicated facts. The book may be warmly recommended to graduate students and researchers in optimization, numerical mathematics, operations research, and other fields which apply optimization methods.

D. Klatte, Zürich