What is continuous optimization ?

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March 11, 2016

Standard format

Finite dimensional continuous optimization problems can be written in the format

$$\min_{x \in \mathbb{R}^n} f(x) \quad \text{s.t.} \quad g_i(x) \leq 0,\ i \in I,\ h_j(x) = 0,\ j \in J$$

with

- continuous decision variables x of dimension $n \in \mathbb{N}$,
- an objective function $f : \mathbb{R}^n \to \mathbb{R}^1$,
- inequality constraint functions $g_i : \mathbb{R}^n \to \mathbb{R}^1$, $i \in I$, with a finite index set I,
- equality constraint functions $h_j : \mathbb{R}^n \to \mathbb{R}^1$, $j \in J$, with a finite index set J.

For applicability of calculus, these functions are often assumed to be at least differentiable or convex.

Below we will extend this format in several ways.

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Finite dimensional continuous optimization

- ‘As opposed to discrete optimization, the variables used in the objective function are required to be continuous variables, that is, to be chosen from a set of real values between which there are no gaps. Because of this continuity assumption, continuous optimization allows the use of calculus techniques.’ (Wikipedia)
- Finite dimensional continuous optimization considers problems with a finite number of continuous decision variables and a finite number of constraints. In contrast, e.g. problems of optimal control search for optimal functions.
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Some applications of Continuous Optimization

- Management problems (cost minimal transportation plans, profit maximization),
- Geometrical problems (optimal sensor placement, waste minimization),
- Mechanical problems (truss topology design),
- Chemical problems (protein folding),
- Statistical problems (parameter fitting, data classification).

Continuous Optimization as a tool for other problems

Finite dimensional continuous optimization problems arise as subproblems ...

- by discretization of the infinite dimensional feasible set in optimal control (thereby giving rise to specially structured, e.g. sparse and/or block-structured, Jacobians and Hessians),
- by discretization of the infinite index set of constraints in semi-infinite optimization,
- by relaxing integrality constraints in integer or mixed-integer optimization problems.

Some important extensions

$$\min_{x\in\mathbb{R}^n} f(x) \quad \text{s.t.} \quad g_i(x)\le 0,\ i\in I,\ h_j(x)=0,\ j\in J$$

- $|I|=+\infty$: semi-infinite optimization (robust optimization, waste minimization),
- $x\in\mathbb{R}^n \ \rightarrow\ (x,y)\in\mathbb{R}^n\times\mathbb{Z}^m$: mixed integer nonlinear optimization (piece good constraints, logic constraints),
- $f:\mathbb{R}^n\to\mathbb{R}^d$ (multi-objective optimization),
- $f \ \rightarrow\ f(t,\cdot)$: parametric optimization (sensitivity analysis, homotopy methods),
- Several coupled parametric problems (Nash games).

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General research questions

- How to quickly identify locally optimal points?
  $\rightarrow$ interior point, SQP, filter methods
- How to identify globally optimal points (at all)?
  $\rightarrow$ branch and bound methods
- How to ensure solvability?
  $\rightarrow$ coercivity checks
- How to do without differentiability or convexity assumptions?
  $\rightarrow$ nonsmooth optimization
- How to estimate the error upon premature termination of solution methods?
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Challenges

In many applications, under weak structural assumptions (i.e. without convexity or differentiability) one needs

- efficient algorithms for the approximation of locally optimal points (with a certificate of optimality – calculus $\leftrightarrow$ heuristics),
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